Answers to Coursebook questions – Chapter 2.6

1 $F_{ave} = \dfrac{\Delta p}{\Delta t} = \dfrac{12.0}{2.00} = 6.00\ \text{N}$.

2 **a** Impulse $= \Delta p = p_{final} - p_{initial} = 0.150 \times (-3.00) - 0.150 \times 3.00 = -0.900\ \text{N s}$.

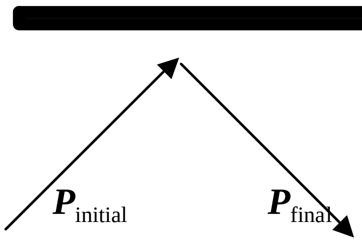
b $|F_{ave}| = \left|\dfrac{\Delta p}{\Delta t}\right| = \dfrac{0.900}{0.125} = 7.20\ \text{N}$. This is the force exerted on the ball by the wall,and so by Newton's third law this is also the force the ball exerted on the wall.

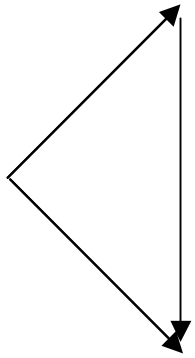
3 The total momentum before the collision is $m \times v + 2m \times (-\dfrac{v}{2}) = 0$. This also the momentum after. If u is the speed after the collision, then $3m \times u = 0 \Rightarrow u = 0$.

4 The total momentum before release is zero. This is also the momentum after release. If u is the required speed, then $2.00 \times v + 4.00 \times 3.50 = 0 \Rightarrow v = -7.00\ \text{m s}^{-1}$ (the minus sign indicates the 2.00 kg mass opposite to the 4.00 kg mass).

5 The person will take $\dfrac{4.00}{v}$ s to get to the front, where v is the speed with respect to the boat. Let u be the speed with which the boat moves backwards. Then from the point of view of an observer at rest in the water the person moves with speed $v - u$. The initial momentum is zero, and so $70.0 \times (v - u) - 200 \times u = 0 \Rightarrow u = \dfrac{70v}{270}$. The distance travelled by the boat is thus $s = ut = \dfrac{70v}{270} \times \dfrac{4.00}{v} = 1.04\ \text{m}$.

6 The change in momentum is given by the following vector diagram. The angle between the vectors is a right angle.





The magnitude of the initial and of the final momentum is $p = 0.250 \times 4.00 = 1.00\ \text{N s}$. The direction of the change of momentum is given in the diagram. Its magnitude is $\sqrt{1.00^2 + 1.00^2} = \sqrt{2.00} = 1.41\ \text{N s}$.

7 **a** $\text{Impulse} = \Delta p = p_{\text{final}} - p_{\text{initial}} = 0.500 \times 4.00 - 0.500 \times (-6.00) = 5.00\ \text{N s}$.

b $|F_{ave}| = \left|\dfrac{\Delta p}{\Delta t}\right| = \dfrac{5.00}{0.200} = 25.0\ \text{N}$. This is the force exerted on the ball by the wall.

8 **a** No, since its speed is increasing.

b It does not apply since there is an external force on the book.

9 **a** There are no external forces acting on the system, and so the momentum must be constant.

b Since the stars rotate, the momentum of each keeps changing direction. This means that the direction of the total momentum also will change, which cannot be true since the direction is constant.

The only way out is to have the stars move at diametrically opposite positions, so that the momenta are equal and opposite. This means that the constant value of the total momentum is in fact zero.

c Since the stars are in diametrically opposite positions all the time, it follows that they have the same period. Since the inner star covers less distance in the same time it has the lower speed. Since the momenta of the two stars are equal it follows that the inner star is also the more massive of the two.

10 **a** Yes, it will move since the fan exerts a force on the air to the right and so, by Newton's third law, the air exerts a force on the barge to the left.

b No, it will not, since the force from **a** will now be met by an equal and opposite force on the sail.

Alternatively, both parts may be answered by using momentum. In **a** there is a net external force on the barge (that from the air on the barge), and so it will move. In **b** the system is closed, there are no external forces and so the momentum cannot change. Since it was zero before the fan was turned on it will remain zero.

11 **a** The earth exerts a force on the person and hence the person exerts the same force upwards on the earth. The earth thus moves up.

b By conservation of momentum $Mv = mu \Rightarrow v = \frac{mu}{M}$. We assume a constant speed u for the falling person equal to the speed acquired after falling 1 m. (This is an estimate!) Hence $u = \sqrt{2gh} = \sqrt{20} \approx 4.5 \text{ m s}^{-1}$. The time taken to fall is $t = \sqrt{\frac{2h}{g}} = \sqrt{2} \approx 1.4 \text{ s}$. Hence, $v = \frac{60 \times 4.5}{6 \times 10^{24}} \approx 4.5 \times 10^{-23} \text{ m s}^{-1}$. The estimated distance moved is then $s = 4.5 \times 10^{-23} \times 1.4 \text{ m} \approx 2 \times 10^{-22} \text{ m}$.

c More, from above.

12 **a** The impulse is the area under the graph and so equals (we use the area of a trapezoid) $\frac{17+7}{2} \times 8.0 = 96 \text{ N s}$.

b Since the impulse is the change in momentum:

$$96 = mv - 0 \Rightarrow v = \frac{96}{3.00} = 32 \text{ m s}^{-1}.$$

c Now, $96 = 0 - mv \Rightarrow v = -\frac{96}{3.00} = -32 \text{ m s}^{-1}$.

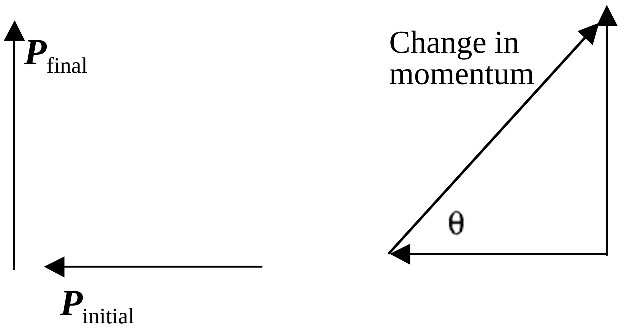
13 $(5000 - m)v = m \times 1500$. In 1 second, $v = 15 \text{ m s}^{-1}$ and so

$$(5000 - m) \times 15 = m \times 1500 \Rightarrow m = \frac{75000}{1515} = 49.5 \text{ kg}.$$

14 The initial total momentum is $4.0 \times 24 - 12.0 \times 2.0 = +72 \text{ N s}$. The final total momentum is $-4.0 \times 3.0 + 12 \times v$. Hence $-12 + 12v = 72 \Rightarrow v = +7.0 \text{ m s}^{-1}$.

15 The initial total momentum is $1200 \times 8.0 - 1400 \times 6.0 = +1200 \text{ N s}$. The final total momentum is $(1200 + 1400) \times v$. Hence, $2600v = 1200 \Rightarrow v = +0.46 \text{ m s}^{-1}$.

16 The change in momentum is found from the diagram:



The initial and final momenta have magnitudes $0.350 \times 8.0 = 2.8\ \mathrm{N\,s}$ and $0.350 \times 12.0 = 4.2\ \mathrm{N\,s}$, respectively.

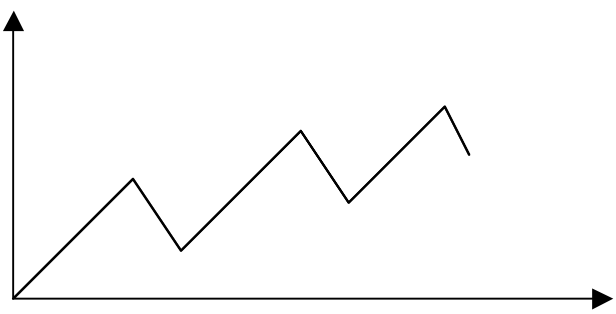
Hence, $\Delta p = \sqrt{2.8^2 + 4.2^2} = 0.350 \times 12.0 = 5.0\ \mathrm{N\,s}$. The direction is given by the angle θ which is $\theta = \tan^{-1}\dfrac{4.2}{2.8} = 56^\circ$.

17 The momentum after the collision is $(1200 + 1300) \times 5.0 = 12{,}500\ \mathrm{N\,s}$. Its components along the axes are the same, and each equals $12{,}500 \times \dfrac{\sqrt{2}}{2} = 8339\ \mathrm{N\,s}$. Then applying conservation of momentum in each direction separately we get:

$1200 \times v_x = 8839 \Rightarrow v_x = 7.4\ \mathrm{m\,s^{-1}}$ and $1300 \times v_y = 8839 \Rightarrow v_y = 6.8\ \mathrm{m\,s^{-1}}$.

18 **a** The impulse supplied to the system is (area under curve) $3 \times 0.5 \times 100 - 25 \times 4 = 50\ \mathrm{N\,s}$. This is the change in momentum i.e. $25 \times \Delta v = 50\ \mathrm{N\,s} \Rightarrow \Delta v = 2.0\ \mathrm{m\,s^{-1}}$.

b Something like (you can easily work out the numbers and slopes on the axes)



19 **a** From the horizontal part of the graph we deduce that the weight of the student is 500 N and so the mass is 50 kg.

b At 0.6 s the reading of the sensor is $R = 1250\,\text{N}$ and since $R - mg = ma$ it follows that $a = \frac{750}{50} = 15\ \text{m s}^{-2}$.

c The student leaves the plate when the reading of the sensor becomes zero, i.e. at 0.8 s.

d The impulse delivered to the student is the area under the **net force** versus time graph. The net force is the reading of the sensor minus the weight. Hence the area (it is enough to concentrate after the 0.5 s point) is: $\frac{1}{2}(0.775 - 0.500) \times 1500 - \frac{1}{2}(0.800 - 0.775) \times 500 = 200\ \text{N s}$. The velocity with which the student leaves the sensor is therefore $v = \frac{200}{50} = 4.0\ \text{m s}^{-1}$ and so the height reached is found using $mgh = \frac{1}{2}mv^2 \Rightarrow h = \frac{v^2}{2g} = 0.8\ \text{m}$.

e Possibilities include replacing the sharp corners with smooth curves. In addition, the student probably accelerates **downwards** as the knees are bent and, so right before the 0.5 s mark, the reading of the sensor will probably be less than 500 N.

20 **a** The ball is dropped from a height of h_1 so (applying conservation of energy) its speed right before impact will be given by $mgh_1 = \frac{1}{2}mv_1{}^2 \Rightarrow v_1 = \sqrt{2gh_1}$. The ball will leave the floor on its way up with a speed found in the same way: $\frac{1}{2}mv_2{}^2 = mgh_2 \Rightarrow v_2 = \sqrt{2gh_2}$. The change in momentum is therefore $mv_2 - (-mv_1) = m(\sqrt{2gh_2} + \sqrt{2gh_1})$ and hence the net average force is $\frac{\Delta p}{\Delta t} = m\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\tau}$.

b $F = 0.250 \times \frac{\sqrt{2 \times 9.81 \times 6.0} + \sqrt{2 \times 9.81 \times 8.0}}{0.125} = 46.8 \approx 47\ \text{N}$. This is the average net force on the ball. The forces on the ball are the reaction from the floor R and its weight, so $R - mg = F \Rightarrow R = mg + F = 46.8 + 0.250 \times 9.81 = 49.2 \approx 49\ \text{N}$. By Newton's third law this is also the force on the floor exerted by the ball.

21 The question assumes the ball hits normally. From the previous problem the net force is $\frac{\Delta p}{\Delta t} = m\frac{v_2 - (-v_1)}{\tau} = m\frac{v_2 + v_1}{\tau}$. This equals $R - mg$, where R is the average force exerted on the ball by the floor. Hence $R = m\frac{v_2 + v_1}{\tau} + mg$.

22 **a** The acceleration is found from $a = \frac{F}{m} = \frac{10}{4.0} = 2.5 \text{ m s}^{-2}$.

b The area under the curve up to 5 s is 50 N s and so

$4.0 \times v = 50 \text{ N s} \Rightarrow v = \frac{50}{4.0} = 12.5 \approx 12 \text{ m s}^{-1}$.

c The acceleration at 8 s is $a = \frac{F}{m} = \frac{4.0}{4.0} = 1.0 \text{ m s}^{-2}$.

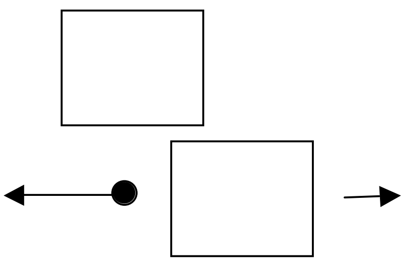
d The total area up to 10 s is 75 N s and so

$4.0 \times v = 75 \text{ N s} \Rightarrow v = \frac{75}{4.0} = 18.75 \approx 19 \text{ m s}^{-1}$.

23 **a** The problem did not make it clear whether the speed of the coins is relative to the astronaut or not. If the speed is relative to the astronaut then:

$60 \times v = 10 \times (\underbrace{5.0 - v}_{\text{speed rel. to ext. observer}}) \Rightarrow v = \frac{50}{70} = 0.714 \approx 0.71 \text{ m s}^{-1}$. The answer given in the book assumed a speed of coins relative to an external observer.

b This is a difficult problem and you should not worry if you cannot follow this solution! It is best to first work generally for this problem. Let us first find the change in velocity of the astronaut when one single coin is thrown.



After the first coin is thrown, with speed u, **relative to the astronaut**, where $u = 5.0 \text{ m s}^{-1}$, we have $(M-m)\Delta v_1 - m(u - \Delta v_1) = 0 \Rightarrow M\Delta v_1 = mu \Rightarrow \Delta v_1 = \frac{mu}{M}$.

This gives the increase in velocity of the astronaut when one coin of mass m is thrown away. Notice that we would get the same result for the increase in velocity even if the astronaut initial had some velocity v_0. In that case we would get from momentum conservation (terms in the same colour cancel out)

$$(M-m)(v_0 + \Delta v_1) - m(u - v_0 - \Delta v_1) = Mv_0$$

$$Mv_0 + M\Delta v_1 - mv_0 - m\Delta v_1 - mu + mv_0 + m\Delta v_1 = Mv_0$$

$$\Delta v_1 = \frac{mu}{M}$$

So we are in position to get the total increase in the speed of the astronaut as the coins are thrown one at a time. The mass of the astronaut when he has *n* coins on him is $M_n = 60 + nm$ where *n* is an integer and *m* = 0.1 kg.

At the beginning, $n = 100$ and so $M_{100} = 70$ kg . When all coins are thrown, $n = 0$, and the mass of the astronaut is just 60 kg.

So after throwing the first coin, the increase in velocity is $\Delta v_1 = \frac{mu}{M_{100}}$.

After throwing the second coin the additional increase in velocity is $\Delta v_2 = \frac{mu}{M_{99}}$ and so on. After throwing away the 100th coin, the increase in velocity will be $\Delta v_{100} = \frac{mu}{M_1}$.

The total increase in velocity is therefore

$$\begin{aligned}\Delta v_1 + \Delta v_2 + \cdots + \Delta v_{100} &= \frac{mu}{M_{100}} + \frac{mu}{M_{99}} + \cdots + \frac{mu}{M_1} = 0.10 \times 5.0 \times \left(\frac{1}{70} + \frac{1}{69.9} + \cdots + \frac{1}{60.1} \right) \\ &= 0.770 \approx 0.77 \text{ m s}^{-1}\end{aligned}$$

This is is larger than when all the coins are thrown together. (This shows the advantage of multiple stage rockets.)

We can also solve this problem by assuming that this is a rocket problem. The coins are the gases leaving the rocket at some fixed rate of μ kg s^{-1} with speed *u* **relative to the rocket**. Let the mass of the rocket be *M* and its speed *v* at time *t*. Then the law of conservation of momentum says that after a mass δm is thrown away the speed will increase to $v + \delta v$ such that ($u - v - \delta v$ is the speed of the ages relative to an external observer – the law of conservation of momentum must be applied from the point of view of a non accelerating external observer)

$$(M - \delta m)(v + \delta v) - \delta m(u - v - \delta v) = Mv \,.$$

This simplifies to

$$Mv + M\delta v - v\delta m - \underbrace{\delta m \delta v}_{\text{neglect-too small}} - u\delta m + v\delta m + \underbrace{\delta m \delta v}_{\text{neglect-too small}} = +Mv$$

$$M\delta v = u\delta m$$

$$M\delta v = -u\delta M$$

$$dv = -u\frac{dM}{M}$$

$$\int_0^0 dv = -u\int_{M_0}^{M} \frac{dM}{M}$$

$$v = -u\ln\frac{M}{M_0} = u\ln\frac{M_0}{M}$$

When $\mu t = 10$ kg all the coins would be thrown away, and so

$$v = 5.0 \ln \frac{70}{60} = 0.77 \text{ m s}^{-1}.$$

24 **a** From 0.5 s to 1.5 s, i.e. for 1 s.

b A rough approximation would be to treat the area a triangle (of area $\frac{1}{2} \times 1.0 \times 120 = 60 \text{ N s}$) but this is too rough and would not be acceptable in a exam. There are roughly 120 rectangles in the area and each has area $0.1 \times 4.0 = 0.4 \text{ N s}$ so that the total area is $0.4 \times 120 = 48 \approx 50 \text{ N s}$.

c From $F_{\text{average}} \Delta t = \Delta p$ we thus find $F_{\text{average}} = 50 \text{ N}$.

